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B.Tech. Degree I Semester Regular/Supplementary Examination in Marine Engineering December 2021

19-208-0101 ENGINEERING MATHEMATICS I

(2019 Scheme)

Time: 3 Hours Maximum Marks: 60

$(5 \times 15 = 75)$

I. (a) Show that the line $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{\left(a^2 - b^2\right)^2}{n^2}$. (5)

(b) Prove that the equation to the locus of the point of intersection of two normals to the parabola $y^2 = 4ax$ which are perpendicular to each other is the curve $y^2 = a\left(x - 3a\right)$. (5)

(c) If e and e_1 are the eccentricities of the hyperbola and its conjugate, show that $\frac{1}{e^2} + \frac{1}{e_1^2} = 1$. (5)

OR

II. (a) Prove that the product of the perpendicular from any point of a hyperbola to its asymptotes is constant. (5)

(b) Find the equation of the asymptotes of the hyperbola $6x^2 - 7xy - 3y^2 + x + 4y = 0$. (5)

(c) Derive the equation of the chord joining the points "t_1" and "t_2" on the rectangular hyperbola $xy = c^2$. Also, obtain the equation of tangent at "t_1". (5)

III. (a) Evaluate (5)

$$\lim_{x \to \frac{\pi}{2}} \left(\sin x\right)^{\tan x}$$

(b) Find all the asymptotes of the curve $x^2 + 2x^2y + xy^2 - x^2 - xy + 2 = 0$. (5)

(c) Find the radius of curvature at the point "t" of the curve $x = a\left(\cos t + t\sin t\right);\ y = a\left(\sin t - t\cos t\right)$. (5)

OR

IV. (a) Obtain the length of an arch of the cycloid whose equations are $x = a\left(t - \sin t\right),\ y = a\left(1 - \cos t\right)$. (5)

(b) Evaluate (5)

$$\int_0^4 x^3\sqrt{4x - x^2}\, dx.$$

(c) Find the volume of the solid obtained by revolving the cardiod $r = a\left(1 + \cos\theta\right)$ about the initial line. (5)

(P.T.O.)

V. (a) Find the n^{th} derivative of $y = 2e^{ax}\cos^2 x$. (5)

(b) If $V = \left(x^2 + y^2 + z^2\right)^{-1/2}$, prove that V satisfies (5)

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0.$$

(c) If $y = a\cos(\log x) + b\sin(\log x)$, show that (5)

$$x^2 y_{n+2} + (2n+1)x\, y_{n+1} + \left(n^2 + 1\right)y_n = 0.$$

OR

VI. (a) The work that must be done in propel a ship of displacement D for a distance S in time t is proportional to $\frac{S^2 D^{3/2}}{t^2}$. Estimate roughly the percentage change in the work, when the distance is increased by 1%, the time is diminished by 1%, and the displacement of the ship is diminished by 3%. (5)

(b) A rectangular box, open at the top, is to have a volume of 32 cubic feet. What must be the dimensions so that the total surface is minimum? (5)

(c) State Euler's Theorem on Homogeneous functions. Use the result to prove that (5)

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

for $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$.

VII. (a) Change the order of integration in (5)

$$I = \int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\, dy\, dx$$

and hence evaluate the same.

(b) Evaluate $\iint_R xy\, dx\, dy$ over the positive quadrant bounded by the line $\frac{x}{a} + \frac{y}{b} = 1$. (5)

(c) Find the volume of the region bounded by $z = x^2 + y^2$, $z = 0$, $x = -a$, $x = a$ and $y = -a$, $y = a$. (5)

OR

VIII. (a) For any three vectors; $\vec{a}, \vec{b}, \vec{c}$. Prove that (5)

$$\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = z\left[\vec{a}\ \vec{b}\ \vec{c}\right].$$

(b) Three vectors $\vec{a} = 12\hat{i} + 4\hat{j} + 3\hat{k}$, $\vec{b} = 8\hat{i} - 12\hat{j} - 9\hat{k}$ and $\vec{c} = 33\hat{i} - 4\hat{j} - 24\hat{k}$ define a parallelepiped. Evaluate the area of its faces and its volume. (5)

(c) Find the system of reciprocal vectors corresponding to the set of vectors: $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = -\hat{i} + \hat{j} + \hat{k}$. (5)

(Continued to 3)

IX. (a) Find the work done in moving a particle in a force field $\vec{f} = 3x^2\hat{i} + (2xz - y)\,\hat{j} + z\hat{k}$ along the line joining the points $(0, 0, 0)$ and $(2, 1, 3)$. (5)

(b) Evaluate $\iint_S \vec{F}.\vec{n}\, ds$, if $\vec{F} = (x+y)\hat{i} + x\hat{j} + z\hat{k}$ and S is the surface of the cube bounded by the planes $x = 0,\ x = 1;\ y = 0,\ y = 1,\ z = 0$ and $z = 1$. (5)

(c) Evaluate $\oint_C \left(e^{-x}\sin y\, dx + e^{-x}\cos y\, dy\right)$ using Green's Theorem, when C is the rectangle whose vertices are $(0,0)$, $(\pi,0)$, $\left(\pi,\frac{\pi}{2}\right)$, $\left(0,\frac{\pi}{2}\right)$. (5)

OR

X. (a) Compute the divergence and curl of the vector $\vec{f} = xyz\hat{i} + 3x^2y\hat{j} + \left(xz^2 - y^2z\right)\hat{k}$ at $(1, 2, -1)$. (5)

(b) A field $\vec{F}$ is of the form (5)

$$\vec{F} = \left(6xy + z^3\right)\hat{i} + \left(3x^2 - z\right)\hat{j} + \left(3xz^2 - y\right)\hat{k}$$

Show that $\vec{F}$ is a conservative field and find its scalar potential.

(c) Evaluate $\nabla^2\left(\frac{1}{r}\right)$. (5)

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